Supply Chain Management for Efficient Consumer Response Conference
18 - 19 May 2012,
Valahia University of Targoviste, Romania
SCM 4 ECR

Bilinear Programming for SCM optimization

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Introduction

- *A supply chain may be defined as an integrated process where several business entities such as suppliers, manufacturers, distributors, and retailers work together to plan, coordinate and control the flow of materials, parts, and finished goods from suppliers to customers.*

- *This chain is concerned with two distinct flows: a forward flow of materials and a backward flow of information.*

- *A supply chain contains two basic, integrated processes:*

  a) production planning and inventory management

  b) distribution and logistics processes

- *Many problems in the supply chain management can be formulated as a network flow problem with specified arc cost functions.*

- *Objectives of the problem is to minimize the total cost by satisfying the demand from the available supply.*

- *Bilinear programming is a particular case of quadric programming whose objective function is not convex or concave.*

Bilinear programming

- *A function f (x, y) is called bilinear if it reduces to a linear one by fixing the vector x or y to a particular value.*
- *Bilinear programming has various applications in constrained bi matrix games, Markovian assignment and complimentarily problems.*
- *Many 0–1 integer programs can be formulated as bilinear problems.*
- *Concave piecewise linear network flow problems, fixed charge network flow problems, and multi item dynamic pricing problems, which are very common in the supply chain management, can be also solved using bilinear formulations.*
- *It should be noted that more general convex/non-convex optimization problems can be reduced to a bilinear problem as well.*
- *Despite a variety of different bilinear problems, most of the practical problems involve a bilinear objective function and linear constraints, and theoretical results are derived for those cases.*

General form of bilinear equation

$$\max\left(c^t x + x^t Q y + d^t y\right)$$

s.t.

$$Ax \le a;$$

$$B^t y \le b;$$

$$x \ge 0;$$

$$y \ge 0;$$

where

$$c, d \in R^n$$

$$x, y \in R^n$$

$$Q \in R^{n*n}$$

$$A, B \in R^{m*n}$$

$$a, b \in R^m$$

Classical algorithms

- *Mountain Climbing Procedure*
- *Cutting Plane Algorithm*

- *They do not always lead to a global optimum, but allow a faster resolution*

Decomposition in linear equations

- *A resolution which leads to a global optimum always is the decomposition in linear equations of type min-max.*

- *Bilinear equation can be decomposed into two linear equations, the optimal function can be obtained by solving one of the two.*

$$\max_x \left(c^t x + \min_u b^t u \right)$$

s.t.

$$Ax \leq a$$

$$Bu \geq d + Qx$$

$$x \geq 0$$

$$u \geq 0$$

or

$$\max_y \left(y^t d + \min_v v^t Q \right)$$

s.t.

$$y^t B \leq b$$

$$v^t A \geq c^t + u^t Q$$

$$v \geq 0$$

$$y \geq 0$$

- *In supply chain management, most applications can be formulated as problems of transport and network optimization, in which points of supply, transit and destination are nodes, and distances between nodes are seen as costs.*

- *In a network of goods distribution shall be looking to minimize the transport costs, eliminating the delay times, maximizing the flow of goods.*

- *In terms of costs, they can be:*
  - *fixed costs such as rental vehicles;*
  - *variables, such as costs due to congestion*

- *The cost optimization flow of goods transported in a network where we have fixed and variable costs is a nonlinear programming problem.*

Location – Allocation Problem

- *It give a total of n cities, L_i , i = 1 ... n, located in geographical coordinate points (x_i, y_i)*

- *A company must build a warehouse to serve the L locations so that the sum of acquisition and transportation costs to be minimal.*

- *It is considered that is store a_k units from m different products P_k, k = 1 ... m.*

- They can supply each store L_i with local units of product b_{ik} of the purchase P_k to the price (per unit) of c_{ik} and the transport price t_k (per unit product per unit distance).

- The deposit coordinates are $D(x_0, y_0)$

- If v_{ik} is the quantity of product P_k that is purchased from the cities L_i then:

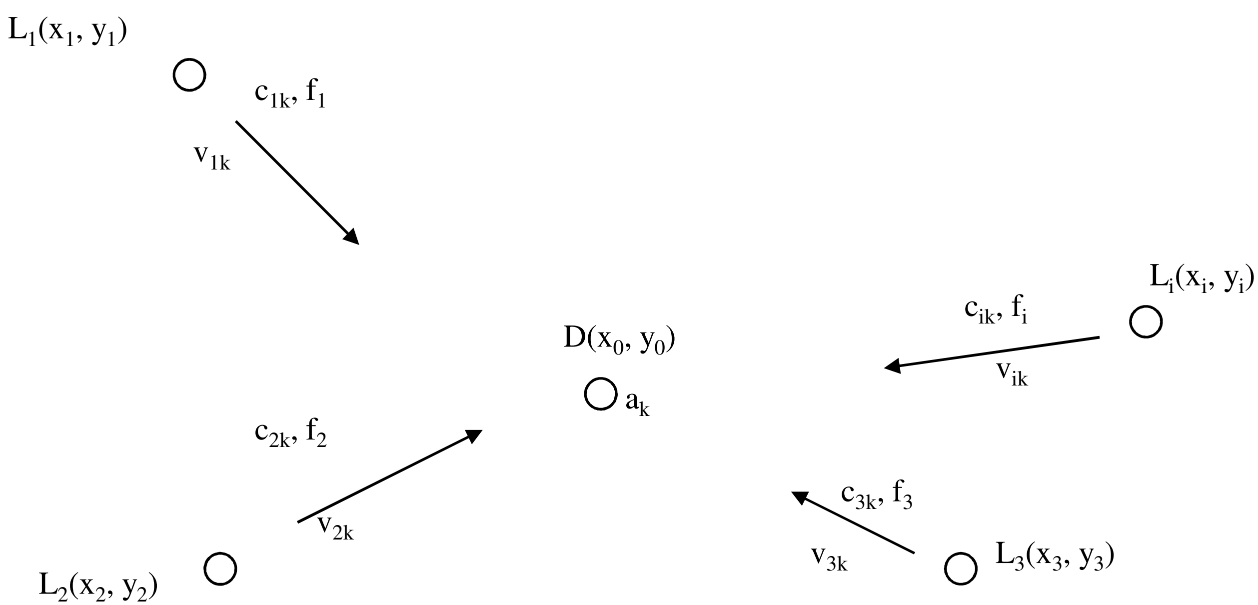

Fig.1 The deposit coordinates

$$\sum_{i=1}^{n} v_{ik} \geq a_k ; k = 1..m$$

$$o \leq v_{ik} \leq b_{ik} ; i = 1..n$$

The total cost of acquisition is:

$$C_a = \sum_{i=1}^{n} \sum_{k=1}^{m} c_{ik} v_{ik}$$

The total cost of transport is:

$$C_t = \sum_{i=1}^{n} \sum_{k=1}^{m} t_k v_{ik} d\left(L_i , D\right)$$

The Euclidean distance from L_i to D is:

$$d(L_i, D) = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2}$$

The final bilinear program is:

$$\min\left(\sum_{i=1}^{n}\sum_{k=1}^{m} v_{ik}\left(c_{ik} + t_k\sqrt{(x_i - x_0)^2 + (y_i - y_0)^2}\right)\right)$$

s.t. $$\sum_{i=1}^{n} v_{ik} \geq a_k ; k = 1..m$$

$$o \leq v_{ik} \leq b_{ik} ; i = 1..n$$

- *In this article we propose a heuristic algorithm for determining the minimum or maximum of bilinear functions.*

- *Search is done sequentially, starting from mid-range points that determine the region of possible values.*

Description of the algorithm for a minimum problem

1. *Is determined the allowable region for X and Y values in equations restrictions, where X = [x_1, x_2, ..., x_n] and Y = [y_1, y_2, ..., y_n].*
2. *In case the vectors X and Y are very high, are divided into equal intervals, for each interval calculating max/min function, then the string values that can be chosen the max/min as global optimum.*
3. *For the initial variables, are chosen the values from mid range, denoted x_c, y_c, i = n/2, j = n/2.*
4. *Is calculated the value of the objective function:*
   - $F_l = f(x_i\text{-}1, y_c)$
   - $F_c = f(x_i, y_c)$
   - $F_u = f(x_{i+1}, y_c)$
5. *Is retained the value of x, denoted x_m, of the function with the lowest value*

5. *Is calculated the value of the objective functions:*

- $F_l = f(x_c, y_{j-1})$
- $F_c = f(x_c, y_j)$
- $F_u = f(x_c, y_{j+1})$

6. *Is retained the value of y, denoted ym, of the function with the lowest value*

7. *If*

$f(x_m, y_{j-1}) > f(x_m, y_m) < f(x_m, y_{j+1})$ *and*

$f(x_{i-1}, y_m) > f(x_m, y_m) < f(x_{i+1}, y_m)$ *then*

the value $f(x_m, y_m)$ *is the minimum function for xm, ym and the program ends.*

8. *Else if* $i, j \neq 1$ *si* $i,j \neq n$

if $f(x_m,y_{j-1}) < f(x_m,y_{j+1})$ *then* $y_c = y_{j-1}$, $j = j-1$

else $y_c = y_{j+1}$, $j = j+1$

$f(x_{i-1},y_m) < f(x_{i+1},y_m)$ *then* $x_c = x_{i-1}$,

$i = i-1$ *else* $x_c = x_{i+1}$, $i = i+1$

9. *Go to step 3*

10. *If* $i, j = 1$ *or* $i,j = n$ *the minimum function is the last value calculated for* x_m,y_m

Conclusions

- *The bilinear programming is applied in particular cases involving two variables and the problem is not convex or concave*
- *The presented algorithm approaches a procedure which can lock into a local optimum, in order to ensure that is always reaches a global optimum used a strict procedure in the sense that the vectors X and Y is dealt in several intervals.*
- *If there are multiple ranges, choose the max / min overall highest value, ie the smallest of the values calculated for each interval.*
- *Size of new obtained ranges should not be too high, because it may have more points max / min.*
- *It's useful if is looking for a local optimum*
- *In case of problems with two variables is easier to formulate the problem as bilinear program then decomposes into linear equations*